The Probabilities of the Mega Millions lottery (http://www.megamillions.com/)

Lat night (Friday 7 March 2007), two winning tickets turned up for a record lottery purse of around $370 million for the jackpot. I played 15 numbers and won $3 with one of them. Yes, the casino kicked my butt.

For each play in the Mega Millions, one selects 5 numbers from among 1 through 56 and one more number, called the mega ball, from among numbers 1 through 42.

The wining configurations are these:

(1) 5+1: jackpot;
(2) 5+0: $250,000 prize
(3) 4+1: $10,000 prize
(4) 4+0: $150 prize
(5) 3+1: $150 prize
(6) 2+1: $10
(7) 3+0: $7
(8) 1+1: $3
(9) 0+1: $2

In the vernacular of probability, saying that the odds of an event are 1 **in** 10 (or 1 **to** 9, sometimes written as 1:9) is inseparably saying that the probability of the event is 1/10, for example. In general, for an event probability p and odds "n **in** m", or "n **to** (m-n)," or "n:(m-n)," $p \equiv n/m$, with m, n real numbers, $n \geq 0$, $m > 0$, and $n \leq m$.

(1, 5+1) The odds of the jackpot is 1 in 175,711,536 and is uncomplicated. The odds is 1 out of the product of two binomial coefficients: (1) choosing 5 distinct numbers from among 56 (1 through 56) distinct numbers and (2) choosing 1 distinct number from among 46 distinct numbers (1 through 46). The first binomial coefficient is $\frac{56!}{5!\,51!}$ and the second is $\frac{46!}{1!\,45!}$. Therefore, the number of possible outcomes for the jackpot is:

$$175{,}711{,}536 = \frac{56!}{5!\,51!} * \frac{46!}{1!\,45!} = \frac{56*55*54*53*52}{5*4*3*2} * 46$$

The odds of any remaining prize is 1 in the number of possible outcomes for the prize. The number of possible outcomes for each of the remaining prizes is a subset of the possible outcomes for the jackpot because the lottery draws only one set of 5+1 numbers on which all smaller prizes depend.

(2, 5+0) Missing the mega ball means that one chose 0 numbers from the single winning number and 1 number from the 45 non-winning numbers, namely $\frac{1!}{0!\,1!} * \frac{45!}{1!\,44!} = 45$.

Therefore, the number of 5+0 outcomes is

$$3{,}904{,}701 = \frac{175{,}711{,}536}{45}$$

(3, 4+1) The divisor that accounts for missing one number from among the first 5 winning numbers is equal to the product of (a) the number of ways in which one can exclude one number out of 5, namely $\frac{5!}{1!\,4!} = 5$, and (b) the number of ways in which one can select that same number from the 51 non-winning numbers, namely $\frac{51!}{1!\,50!} = 51$.

Therefore, the number of 4+1 outcomes is

$$689{,}065 = \frac{175{,}711{,}536}{5*51} = \frac{175{,}711{,}536}{255}$$

(4, 4+0) The divisor that accounts for missing one number from among the first 5 winning numbers and for missing the mega ball is:
(a) the number of ways in which one can exclude one number out of the first 5, namely 5! / (1! 4!) = 5; times
(b) the number of ways in which one can select the missed number from among the 51 non-winning numbers; 51! / (1! 50!) = 51; times
(c) the number of ways in which one misses the mega ball from the winning mega ball, namely 1! / (0! 1!) = 1; times
(d) the number of ways in which one can select the non-winning mega ball from among the 45 non-winning mega balls, 45! / (1! 44!). = 45.
Therefore, the number of 4+0 outcomes is

$$15{,}313 = \frac{175{,}711{,}536}{5*51*1*45} = \frac{175{,}711{,}536}{11{,}475}$$

(5, 3+1) The divisor that accounts for the 2 missed numbers from among the first five winning numbers is:
(a) the number of ways in which one can miss 2 numbers from among the first 5 winning numbers, 5! / (2! 3!) = 10; times
(b) the number of ways in which one can select the 2 missed numbers from among the non-winning 51 numbers, 51! / (2! 49!) = 1,275.
Therefore, the number of 3+1 outcomes is

$$13{,}781 = \frac{175{,}711{,}536}{10*1{,}275} = \frac{175{,}711{,}536}{12{,}750}$$

(6, 2+1) The divisor that accounts for the 3 missed numbers from among the first 5 winning numbers is:
(a) the number of ways in which one can miss 3 numbers from among the 5 winning numbers, 5! / (3! 2!) = 10; times
(b) the number of ways in which one can select 3 numbers from among the 51 non-winning numbers, 51! / (3! 48!) = 20,825.
Therefore, the number of 2+1 outcomes is

$$844 = \frac{175{,}711{,}536}{10 * 20{,}825} = \frac{175{,}711{,}536}{208{,}250}$$

(7, 3+0) The divisor that accounts for the 2 missed numbers from among the first 5 winning numbers and the missed mega ball is:
(a) the number of ways in which one can miss 2 numbers from among the first 5 winning numbers, 5! / (2! 3!) = 10; times
(b) the number of ways in which one can select the 2 missed numbers from among the non-winning 51 numbers, 51! / (2! 49!) = 1,275; times
(c) the number of ways in which one misses the mega ball from the winning mega ball, namely 1! / (0! 1!) = 1; times
(d) the number of ways in which one can select the non-winning mega ball from among the 45 non-winning mega balls, 45! / (1! 44!). = 45.
Therefore, the number of 3+0 outcomes is

$$306 = \frac{175{,}711{,}536}{10 * 1{,}275 * 1 * 45} = \frac{175{,}711{,}536}{573{,}750}$$

(8, 1+1) The divisor that accounts for the 4 missed numbers from among the first 5 winning numbers is:
(a) the number of ways in which one can miss 4 numbers from among the first 5 winning numbers, 5! / (1! 4!) = 5; times
(b) the number of ways in which one can select 4 numbers from among the 51 non-winning numbers, 51! / (4! 47!) = 249,900.
Therefore, the number of 1+1 outcomes is

$$141 = \frac{175{,}711{,}536}{5 * 249{,}900} = \frac{175{,}711{,}536}{1{,}249{,}500}$$

(9, 0+1) The divisor that accounts for the 5 missed numbers from among the first 5 winning numbers is:
(a) the number of ways in which one can miss 5 numbers from among the first 5 winning numbers, 5! / (0! 5!) = 1; times
(b) the number of ways in which one can select the five non-winning numbers from among the 51 non-winning numbers, 51! / (5! 46!) = 2,349,060.
Therefore, the number of 0+1 outcomes is

$$75 = \frac{175{,}711{,}536}{2{,}349{,}060}$$

The Probabilities of the Power Ball lottery (http://www.powerball.com//)

The lottery named PowerBall, available in the District of Columbia and several states, is similar to MegaMillions but with this prize schedule.

(1) 5+1: Jackpot;
(2) 5+0: $200,000;
(3) 4+1: $10,000
(4) 4+0: $100
(5) 3+1: $100
(6) 3+0: $7
(7) 2+1: $7
(8) 1+1: $4
(9) 0+1: $3

The lottery has a power-play option that doubles the price of each number one buys with that option.

In this lottery, one chooses 5 out of 55 numbers and 1 out of 42 numbers.

So here is the number of winning outcomes for each:

$$(1,\ 5{+}1)\ \ 146{,}107{,}962 = \frac{55!}{5!\,50!} * \frac{42!}{1!\,41!}$$

$$(2,\ 5{+}0)\ \ 3{,}563{,}609 = \frac{55!}{5!\,50!} * \frac{42!}{1!\,41!} \Bigg/ \left(\frac{1!}{1!\,0!} * \frac{41!}{1!\,40!} \right)$$

$$(3,\ 4{+}1)\ \ 584{,}432 = \frac{55!}{5!\,50!} * \frac{42!}{1!\,41!} \Bigg/ \left(\frac{5!}{1!\,4!} * \frac{50!}{1!\,49!} \right)$$

$$(4,\ 4{+}0)\ \ 14{,}254 = \frac{55!}{5!\,50!} * \frac{42!}{1!\,41!} \Bigg/ \left(\frac{5!}{1!\,4!} * \frac{50!}{1!\,49!} * \frac{1!}{1!\,0!} * \frac{41!}{1!\,40!} \right)$$

$$(5,\ 3{+}1)\ \ 11{,}927 = \frac{55!}{5!\,50!} * \frac{42!}{1!\,41!} \Bigg/ \left(\frac{5!}{2!\,3!} * \frac{50!}{2!\,48!} \right)$$

$$(6,\ 3{+}0)\ \ 291 = \frac{55!}{5!\,50!} * \frac{42!}{1!\,41!} \Bigg/ \left(\frac{5!}{2!\,3!} * \frac{50!}{2!\,48!} * \frac{1!}{1!\,0!} * \frac{41!}{1!\,40!} \right)$$

$$(7,\ 2{+}1)\ \ 745 = \frac{55!}{5!\,50!} * \frac{42!}{1!\,41!} \Bigg/ \left(\frac{5!}{2!\,3!} * \frac{50!}{3!\,47!} \right)$$

$$(8,\ 1{+}1)\ \ 127 = \frac{55!}{5!\,50!} * \frac{42!}{1!\,41!} \Bigg/ \left(\frac{5!}{1!\,4!} * \frac{50!}{4!\,46!} \right)$$

$$(9,\ 0{+}1)\ \ 69 = \frac{55!}{5!\,50!} * \frac{42!}{1!\,41!} \Bigg/ \left(\frac{5!}{0!\,5!} * \frac{50!}{5!\,45!} \right)$$