

The Probabilities of the Mega Millions lottery (<http://www.megamillions.com/>)

Last night (Friday 7 March 2007), two winning tickets turned up for a record lottery purse of around \$370 million for the jackpot. I played 15 numbers and won \$3 with one of them. Yes, the casino kicked my butt.

For each play in the Mega Millions, one selects 5 numbers from among 1 through 56 and one more number, called the mega ball, from among numbers 1 through 42.

The winning configurations are these:

- (1) 5+1: jackpot;
- (2) 5+0: \$250,000 prize
- (3) 4+1: \$10,000 prize
- (4) 4+0: \$150 prize
- (5) 3+1: \$150 prize
- (6) 2+1: \$10
- (7) 3+0: \$7
- (8) 1+1: \$3
- (9) 0+1: \$2

In the vernacular of probability, saying that the odds of an event are 1 **in** 10 (or 1 **to** 9, sometimes written as 1:9) is inseparably saying that the probability of the event is 1/10, for example. In general, for an event probability p and odds “ n **in** m ”, or “ n **to** $(m-n)$,” or “ $n:(m-n)$,” $p \equiv n/m$, with m, n real numbers, $n \geq 0$, $m > 0$, and $n \leq m$.

(1, 5+1) The odds of the jackpot is 1 in 175,711,536 and is uncomplicated. The odds is 1 out of the product of two binomial coefficients: (1) choosing 5 distinct numbers from among 56 (1 through 56) distinct numbers and (2) choosing 1 distinct number from among 46 distinct numbers (1 through 46). The first binomial coefficient is $\frac{56!}{5!51!}$ and the second is $\frac{46!}{1!45!}$. Therefore, the number of possible outcomes for the jackpot is:

$$175,711,536 = \frac{56!}{5!51!} * \frac{46!}{1!45!} = \frac{56 * 55 * 54 * 53 * 52}{5 * 4 * 3 * 2} * 46$$

The odds of any remaining prize is 1 in the number of possible outcomes for the prize. The number of possible outcomes for each of the remaining prizes is a subset of the possible outcomes for the jackpot because the lottery draws only one set of 5+1 numbers on which all smaller prizes depend.

(2, 5+0) Missing the mega ball means that one chose 0 numbers from the single winning number and 1 number from the 45 non-winning numbers, namely $\frac{1!}{0!1!} * \frac{45!}{1!44!} = 45$.

Therefore, the number of 5+0 outcomes is

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$$3,904,701 = \frac{175,711,536}{45}$$

(3, 4+1) The divisor that accounts for missing one number from among the first 5 winning numbers is equal to the product of (a) the number of ways in which one can exclude one number out of 5, namely $\frac{5!}{1! 4!} = 5$, and (b) the number of ways in which one can select that same number from the 51 non-winning numbers, namely $\frac{51!}{1! 50!} = 51$.

Therefore, the number of 4+1 outcomes is

$$689,065 = \frac{175,711,536}{5 * 51} = \frac{175,711,536}{255}$$

(4, 4+0) The divisor that accounts for missing one number from among the first 5 winning numbers and for missing the mega ball is:
(a) the number of ways in which one can exclude one number out of the first 5, namely $5! / (1! 4!) = 5$; times
(b) the number of ways in which one can select the missed number from among the 51 non-winning numbers; $51! / (1! 50!) = 51$; times
(c) the number of ways in which one misses the mega ball from the winning mega ball, namely $1! / (0! 1!) = 1$; times
(d) the number of ways in which one can select the non-winning mega ball from among the 45 non-winning mega balls, $45! / (1! 44!) = 45$.

Therefore, the number of 4+0 outcomes is

$$15,313 = \frac{175,711,536}{5 * 51 * 1 * 45} = \frac{175,711,536}{11,475}$$

(5, 3+1) The divisor that accounts for the 2 missed numbers from among the first five winning numbers is:

(a) the number of ways in which one can miss 2 numbers from among the first 5 winning numbers, $5! / (2! 3!) = 10$; times

(b) the number of ways in which one can select the 2 missed numbers from among the non-winning 51 numbers, $51! / (2! 49!) = 1,275$.

Therefore, the number of 3+1 outcomes is

$$13,781 = \frac{175,711,536}{10 * 1,275} = \frac{175,711,536}{12,750}$$

(6, 2+1) The divisor that accounts for the 3 missed numbers from among the first 5 winning numbers is:

(a) the number of ways in which one can miss 3 numbers from among the 5 winning numbers, $5! / (3! 2!) = 10$; times

(b) the number of ways in which one can select 3 numbers from among the 51 non-winning numbers, $51! / (3! 48!) = 20,825$.

Therefore, the number of 2+1 outcomes is

$$844 = \frac{175,711,536}{10 * 20,825} = \frac{175,711,536}{208,250}$$

(7, 3+0) The divisor that accounts for the 2 missed numbers from among the first 5 winning numbers and the missed mega ball is:

(a) the number of ways in which one can miss 2 numbers from among the first 5 winning numbers, $5! / (2! 3!) = 10$; times

(b) the number of ways in which one can select the 2 missed numbers from among the non-winning 51 numbers, $51! / (2! 49!) = 1,275$; times

(c) the number of ways in which one misses the mega ball from the winning mega ball, namely $1! / (0! 1!) = 1$; times

(d) the number of ways in which one can select the non-winning mega ball from among the 45 non-winning mega balls, $45! / (1! 44!) = 45$.

Therefore, the number of 3+0 outcomes is

$$306 = \frac{175,711,536}{10 * 1,275 * 1 * 45} = \frac{175,711,536}{573,750}$$

(8, 1+1) The divisor that accounts for the 4 missed numbers from among the first 5 winning numbers is:

(a) the number of ways in which one can miss 4 numbers from among the first 5 winning numbers, $5! / (1! 4!) = 5$; times

(b) the number of ways in which one can select 4 numbers from among the 51 non-winning numbers, $51! / (4! 47!) = 249,900$.

Therefore, the number of 1+1 outcomes is

$$141 = \frac{175,711,536}{5 * 249,900} = \frac{175,711,536}{1,249,500}$$

(9, 0+1) The divisor that accounts for the 5 missed numbers from among the first 5 winning numbers is:

(a) the number of ways in which one can miss 5 numbers from among the first 5 winning numbers, $5! / (0! 5!) = 1$; times

(b) the number of ways in which one can select the five non-winning numbers from among the 51 non-winning numbers, $51! / (5! 46!) = 2,349,060$.

Therefore, the number of 0+1 outcomes is

$$75 = \frac{175,711,536}{2,349,060}$$

The Probabilities of the Power Ball lottery (<http://www.powerball.com/>)

The lottery named PowerBall, available in the District of Columbia and several states, is similar to MegaMillions but with this prize schedule.

- (1) 5+1: Jackpot;
- (2) 5+0: \$200,000;
- (3) 4+1: \$10,000
- (4) 4+0: \$100
- (5) 3+1: \$100
- (6) 3+0: \$7
- (7) 2+1: \$7
- (8) 1+1: \$4
- (9) 0+1: \$3

The lottery has a power-play option that doubles the price of each number one buys with that option.

In this lottery, one chooses 5 out of 55 numbers and 1 out of 42 numbers.

So here is the number of winning outcomes for each:

$$(1, 5+1) 146,107,962 = \frac{55!}{5! 50!} * \frac{42!}{1! 41!}$$

$$(2, 5+0) 3,563,609 = \frac{55!}{5! 50!} * \frac{42!}{1! 41!} / \left(\frac{1!}{1! 0!} * \frac{41!}{1! 40!} \right)$$

$$(3, 4+1) 584,432 = \frac{55!}{5! 50!} * \frac{42!}{1! 41!} / \left(\frac{5!}{1! 4!} * \frac{50!}{1! 49!} \right)$$

$$(4, 4+0) 14,254 = \frac{55!}{5! 50!} * \frac{42!}{1! 41!} / \left(\frac{5!}{1! 4!} * \frac{50!}{1! 49!} * \frac{1!}{1! 0!} * \frac{41!}{1! 40!} \right)$$

$$(5, 3+1) 11,927 = \frac{55!}{5! 50!} * \frac{42!}{1! 41!} / \left(\frac{5!}{2! 3!} * \frac{50!}{2! 48!} \right)$$

$$(6, 3+0) 291 = \frac{55!}{5! 50!} * \frac{42!}{1! 41!} / \left(\frac{5!}{2! 3!} * \frac{50!}{2! 48!} * \frac{1!}{1! 0!} * \frac{41!}{1! 40!} \right)$$

$$(7, 2+1) 745 = \frac{55!}{5! 50!} * \frac{42!}{1! 41!} / \left(\frac{5!}{2! 3!} * \frac{50!}{3! 47!} \right)$$

$$(8, 1+1) 127 = \frac{55!}{5! 50!} * \frac{42!}{1! 41!} / \left(\frac{5!}{1! 4!} * \frac{50!}{4! 46!} \right)$$

$$(9, 0+1) 69 = \frac{55!}{5! 50!} * \frac{42!}{1! 41!} / \left(\frac{5!}{0! 5!} * \frac{50!}{5! 45!} \right)$$