$$P_0 = \frac{-U_0}{(1+y)^n}\left[1 + c\frac{(1+y)^n - 1}{y}\right] \equiv -U_0 * PVF(y,n) * [1 + c * FVFA(y,n)]$$

$P_0 > 0$: Price at t = 0, time of security settlement; sign convention: short position.

$\mathrm{U}_0 < 0$: Face value at security settlement (t = 0) due n periods later (t = n); sign convention: short position.

c > 0: Contracted rate of interest paid periodically on the base $-U_0$.

y > 0: Settled periodic yield rate (market or privately agreed periodic yield rate).

$\mathrm{PVF(y,n)} = (1+y)^{-n}$: Present value (at t = 0) of $1 received n (t = n) periods later. Finance vernacular refers to this as “present value factor of $1 at yield y received in n periods,” or in some similar phrase.

$\mathrm{FVFA(y,n)} = \sum_{i=0}^{n}(1+y)^i \equiv \frac{(1+y)^n - 1}{y}$: Future value factor of an ordinary n-period annuity discounted at rate y. Definitional divergence: a term equivalent to ordinary n-period annuity is “an n-period annuity in arrears.”

$P_i \in \{\mathrm{P}_0, \mathrm{P}_1, \mathrm{P}_2, \cdots \mathrm{P}_n \equiv -\mathrm{U}_0\}$: Unamortized basis of the debt the end of the ith period. In the United States, when the debtor has tax deductibility of interest expense, the tax deductible interest exists at the end of the n periods t = 1, …, n and the interest amounts are $\mathrm{P}_0 y, \mathrm{P}_1 y, \mathrm{P}_{n-1} y$

The periodic amortization amount for a premium (y < c) sale is negative and for a discount (y > c) is positive. Its value is $P_i - P_{i-1}$ for i = 1, 2, …, n, with the value

$P_i - P_{i-1} = U_0 c + P_{i-1} y$ for i = 1, 2, …, n

At the inception of a debt instrument, the amortized cumulative amount of premium ($P_0+U_0 > 0$ for y < c) or discount ($P_0+U_0 < 0$ for y > c) is 0. That is, $A_0 = 0$. At inception, the not-yet-amortized cumulative amount of premium or discount is

$P_0 + U_0$. That is, $A'_0 \equiv P_0 + U_0$. At any time $0 \le t \le n$ $A_t + A'_t \equiv P_0 + U_0$

The amortized cumulative amount of premium or discount after amortization at the end of period j = 1, 2, …, n is $A_j = jU_0 c + \sum_{i=0}^{j-1} P_i y$

Some algebra leads to the formula for the periodic amortization (Di for i = 1, 2, …, n) and for the period-to-date cumulative periodic amortization (Ai for i = 1, 2, …, n).

Both Ai and Di carry period-independent factor

$$U_0[c - y * PVF(y,n) * [1 + c * FVFA(y,n)]]$$

That period-independent factor happens to be the amortization of the premium or discount at the end of the first period, so defining D1 as that constant is convenient.

$$D_1 \equiv U_0\left[c - y * PVF(y,n) * \left[1 + c * FVFA(y,n)\right]\right] \equiv U_0 c + P_0 y$$

For Di (i = 1, 2, …, n) in general, the formula is this.

$$D_i \equiv D_1 * (1+y)^{i-1} \equiv D_1 * FVF(y, i-1)$$

The period-to-date sum of those Di's includes the common factor D1 and the sum of the factors $(1+y)^{i-1}$ for i = 1, …, j with j representing the period ending the summing. The sum of those factors is FVFA(y,j), so

$$A_i \equiv D_1 * FVFA(y,i)$$

The period-to-date carrying balance P_i is this.

$$P_i \equiv P_0 + A_i \equiv P_0 + \left[U_0 c + P_0 y\right] * FVFA(y,i)$$